TR-934

DEPENDENCE OF Z-PARAMETERS ON THE LF TRANSISTOR T-EQUIVALENT CIRCUIT

Nicholas Kyriakopoulos

5 April 1961

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FOR THE COMMANDER:
APPROVED BY

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ABSTRACT

The Z-parameters of a transistor have been calculated in terms of the transistor T-equivalent circuit parameters. The calculations have been made for a frequency of 1.0 kc. In addition, each of the independent equivalent circuit parameters was halved and doubled while the rest were held constant, and the effect on the Z-parameters investigated.

The results indicate that in the common base and common emitter configurations the input impedance depends on r_b only, while the forward transfer impedance depends on r_b and r_e. The rest of the parameters are functions of r_c and C_c only.

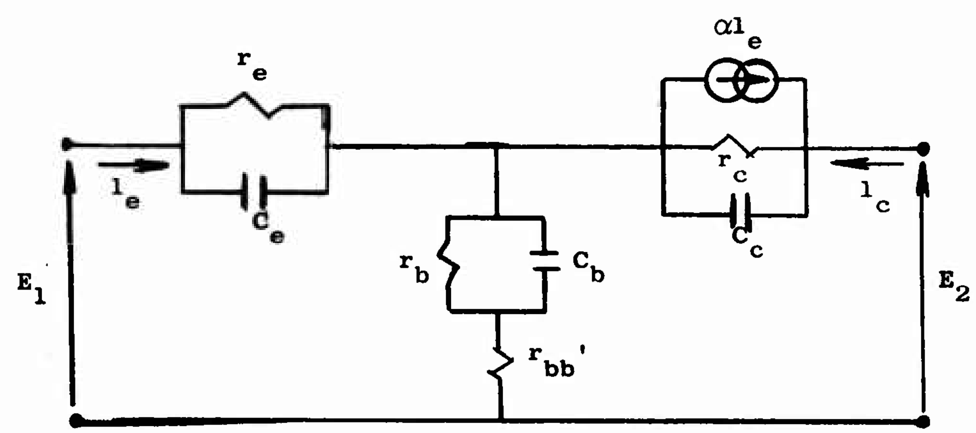
1. INTRODUCTION

The purpose of this report is to investigate the dependence of the terminal characteristics of a transistor on its T-equivalent circuit parameters. This dependence is of primary importance in transistor design, since a given set of terminal characteristics can be achieved by specifying the values of the internal parameters; these values, in turn, are functions of junction area, doping, etc., and can be controlled to a certain extent.

Because of the numerous and tedious calculations involved, this work was performed with the help of an IBM 704 digital computer. This report will include the relations between the terminal characteristics and the equivalent circuit parameters, a brief discussion of the programming techniques used, and the results obtained for common base, common emitter and common collector configurations.

2. DISCUSSION

Of the various transistor equivalent circuits available, the T-equivalent was chosen for the purposes of the present investigation. Although basically a low frequency equivalent circuit, its useful frequency range is increased by including an emitter and base capacitance. On the basis of the circuit illustrated below,



the Z-parameters in the common base, common emitter and common collector configuration are found as follows:

$$Z_{11e} = Z_{11b} = r_{bb'} + \frac{r_b}{1+(\omega C_b r_b)^2} + \frac{r_e}{1+(\omega C_e r_e)^2} - j\omega\left(\frac{C_b r_b^2}{1+(\omega C_b r_b)^2} + \frac{C_e r_e^2}{1+(\omega C_e r_e)^2}\right)$$

$$Z_{12b} = r_{bb'} + \frac{r_b}{1+(\omega C_b r_b)^2} - j\omega \frac{C_b r_b^2}{1+(\omega C_b r_b)^2}$$

$$Z_{21b} = r_{bb'} + \frac{r_b}{1+(\omega C_b r_b)^2} + \frac{\alpha r_c}{1+(\omega C_c r_c)^2} - j\omega\left(\frac{\alpha C_c r_c^2}{1+(\omega C_c r_c)^2} + \frac{C_b r_b^2}{1+(\omega C_b r_b)^2}\right)$$

$$Z_{11_c} = Z_{22b} = r_{bb'} + \frac{r_b}{1+(\omega C_b r_b)^2} + \frac{r_c}{1+(\omega C_c r_c)^2} - j\omega\left(\frac{C_b r_b^2}{1+(\omega C_b r_b)^2} + \frac{C_e r_e^2}{1+(\omega C_c r_c)^2}\right)$$

$$Z_{12e} = \frac{r_e}{1+(\omega C_e r_e)^2} - j\omega \frac{r_e^2 C_e}{1+(\omega C_e r_e)^2}$$

$$Z_{21e} = \frac{r_e}{1+(\omega C_e r_e)^2} - \frac{\alpha r_c}{1+(\omega C_c r_c)^2} + j\omega\left(\frac{\alpha C_c r_c^2}{1+(\omega C_c r_c)^2} - \frac{C_e r_e^2}{1+(\omega C_e r_e)^2}\right)$$

$$Z_{22_c} = Z_{22e} = \frac{r_e}{1+(\omega C_e r_e)^2} + \frac{(1-\alpha) r_c}{1+(\omega C_c r_c)^2} - jw\left(\frac{(1-\alpha) C_c r_c^2}{1+(\omega C_c r_c)^2} + \frac{r_e}{1+(\omega C_e r_e)^2}\right)$$

$$Z_{12c} = \frac{(1-\alpha) r_c}{1+(\omega C_c r_c)^2} - j\omega \frac{(1-\alpha) C_c r_c^2}{1+(\omega C_c r_c)^2}$$

$$Z_{21e} = \frac{r_c}{1+(\omega C_c r_c)^2} - j\omega \frac{C_c r_c^2}{1+(\omega C_c r_c)^2}$$

Thus, the terminal transistor impedances are expressed in terms of the equivalent circuit parameters in the three configurations. For given values of these inherent or intrinsic parameters the impedances can be calculated.

In the present study, the IBM 704 digital computer of the National Bureau of Standards was used to calculate the terminal parameters. The program was written in the FORTRAN (Formula Translation) language. FORTRAN is a system by which a program written in a relatively simple language can be translated into the language that the computer understands. This technique enables a non-professional programmer to use the computer for a wide range of problems with a minimum of difficulty (ref 1).

It was desired to find the effect on the terminal characteristics of the transistor when one of the equivalent circuit parameters was varied while the others were held constant. The data used in this report were taken from the data sheet for a 2N220 transistor. In the equivalent circuit representation C_e was assumed to be very small, so the capacitive reactance in parallel with r_e gave an effective impedance approximately equal to r_e. In view of the low frequency being considered, this appears to be a valid assumption. The values of the equivalent circuit parameters used as basis for the calculation of the terminal characteristics appear in table I. Since C_b was not given in the original

TABLE I

r_{bb}	r_b	C_b	r_e	r_c	C_c	α
ohms	ohms	pf	ohms	megohms	pf	
190	1085	8850	37.7	2.86	50	.985

data, the value of 8850 pf was taken from measurements that were performed on a 2N180 transistor (ref 2).

The values given in table I and the terminal parameters corresponding to these values (table II) were used as bases for normalizing the results. Each value of the equivalent circuit parameters was halved and doubled while the others were kept constant. For each set of values the terminal characteristics were computed at a frequency of 1.00 kc. The results of these computations appear in figures 1 through 13. In the graphs, the ordinate represents the normalized terminal characteristics while the abscissa represents the normalized equivalent circuit parameters.

Table II

$Z_{11e} = 1.31 \times 10^3 - j6.52 \times 10^1$	$Z_{12e} = 3.77 \times 10^1 + j0.0$
$Z_{21e} = -1.60 \times 10^6 + j1.40 \times 10^6$	$Z_{22e} = 2.38 \times 10^4 - j2.13 \times 10^4$
$Z_{11b} = 1.31 \times 10^3 - j6.52 \times 10^1$	$Z_{12b} = 1.27 \times 10^3 - j6.52 \times 10^1$
$Z_{21b} = 1.56 \times 10^6 - j1.40 \times 10^6$	$Z_{22b} = 1.58 \times 10^6 - j1.42 \times 10^6$
$Z_{11c} = 1.58 \times 10^6 - j1.42 \times 10^6$	$Z_{12c} = 2.37 \times 10^4 - j2.13 \times 10^4$
$Z_{21c} = 1.58 \times 10^6 - j1.42 \times 10^6$	$Z_{22c} = 2.38 \times 10^4 - j2.13 \times 10^4$

3. RESULTS

The greatest effect on most of the terminal characteristics of a transistor, at the frequency considered, is brought about by the variation of r_c and C_c. The only exceptions are the parameters with which r_c and C_c are not associated. Thus, in the common emitter and common base configurations the input impedance is largely a function of r_b and C_b while r_{bb} has little effect and r_e plays an insignificant role. In fact, the only terminal characteristic which is affected by r_e is the common emitter forward transfer impedance Z_{12e} which for all practical purposes is directly proportional to r_e. In this investigation, the emitter capacitance was assumed negligible, thus making the forward transfer impedance equal to r_e.

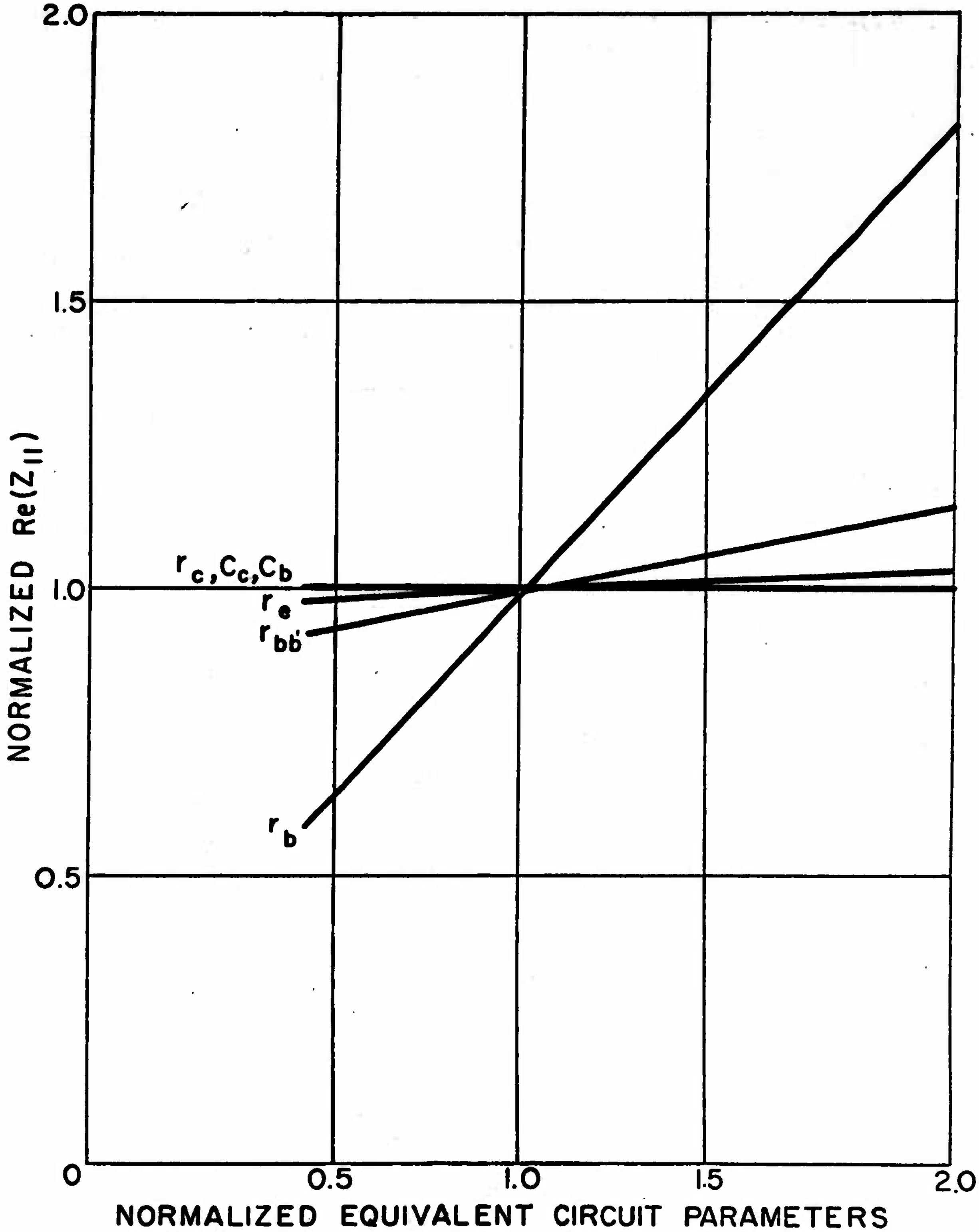


Figure 1. Real part of Z_{11} versus equivalent circuit parameters. Common base, emitter configuration.

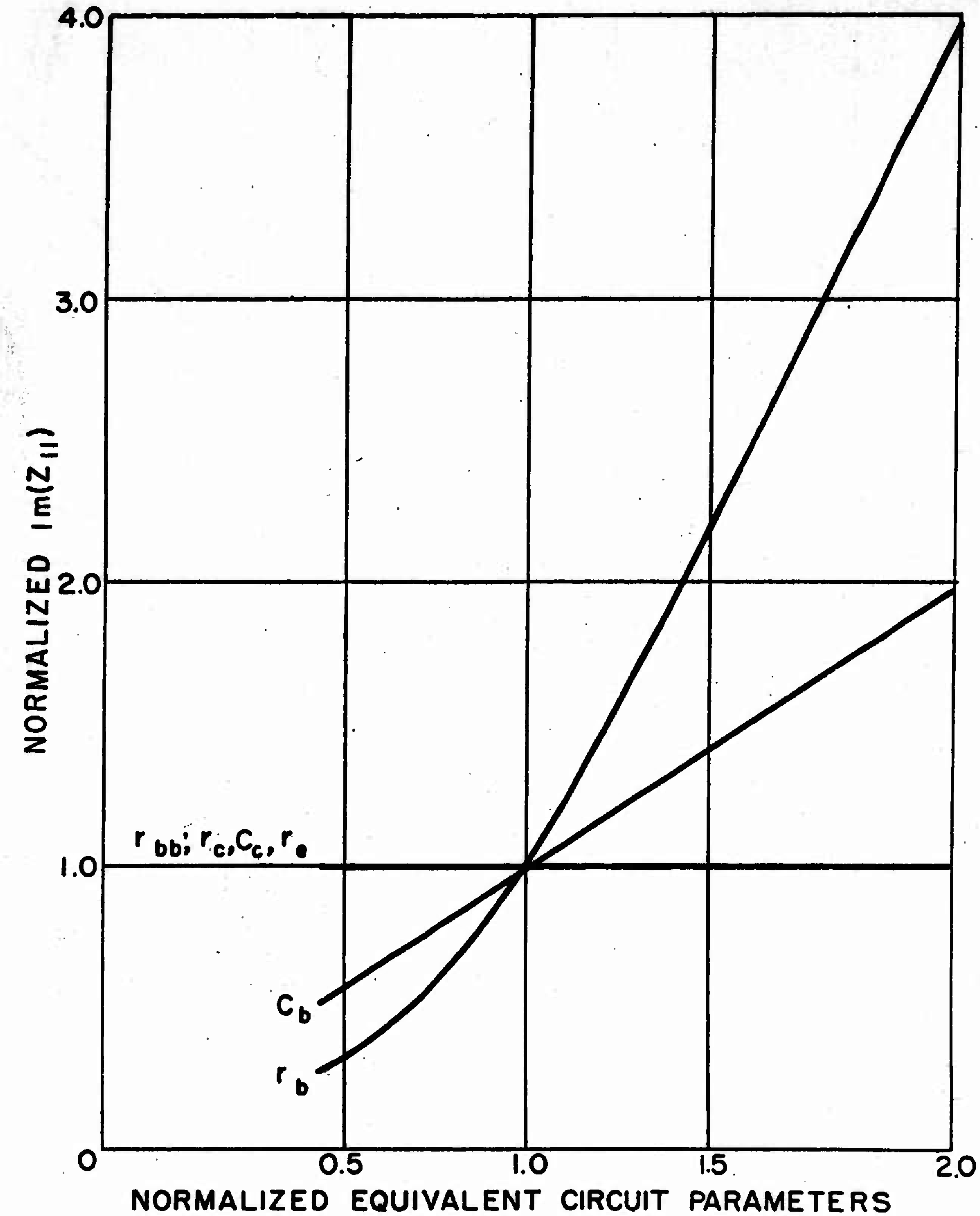


Figure 2. Imaginary part of Z_{11} versus equivalent circuit parameters. Common base, emitter configuration.

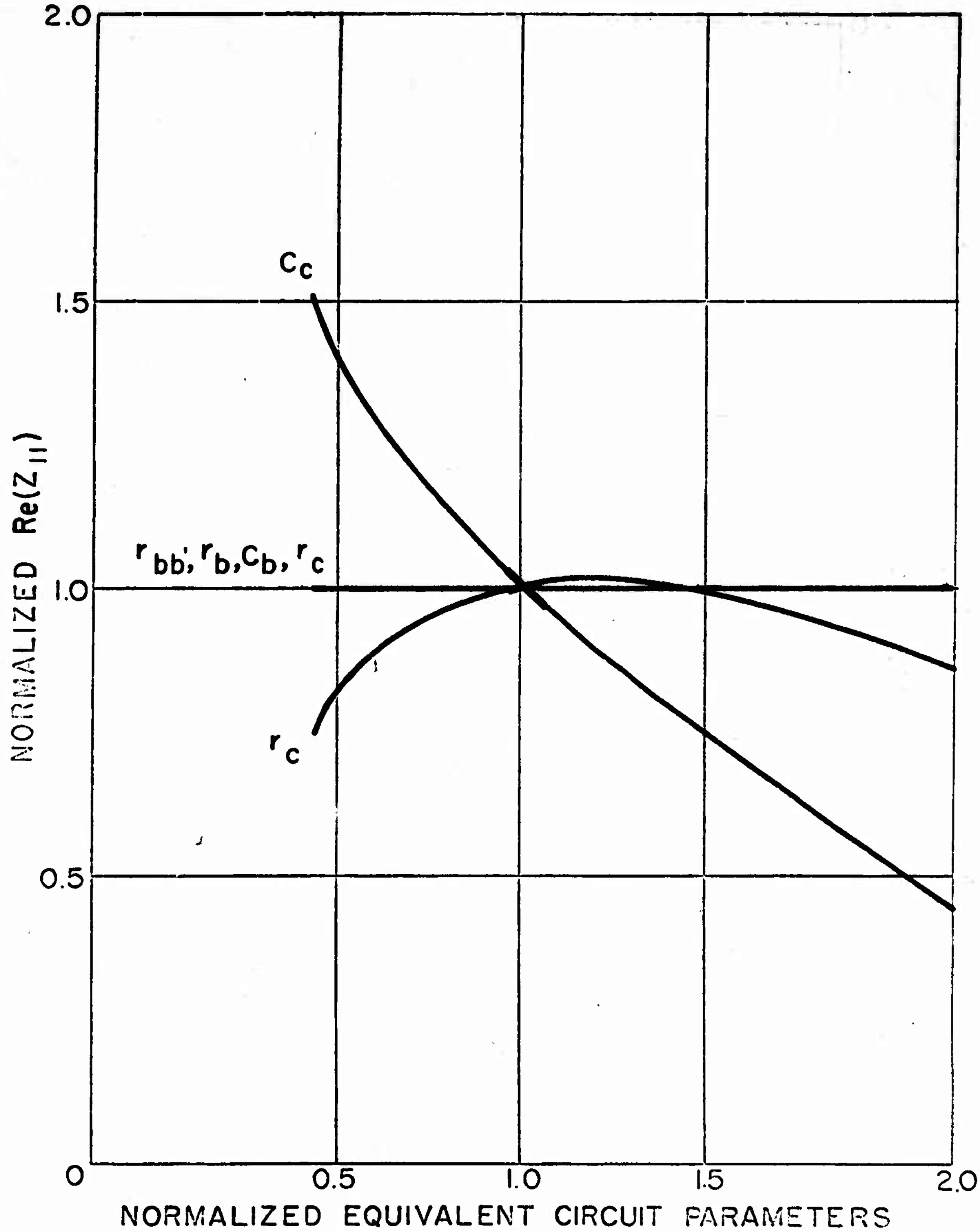


Figure 3. Real part of Z_{11} versus equivalent circuit parameters. Common collector configuration.

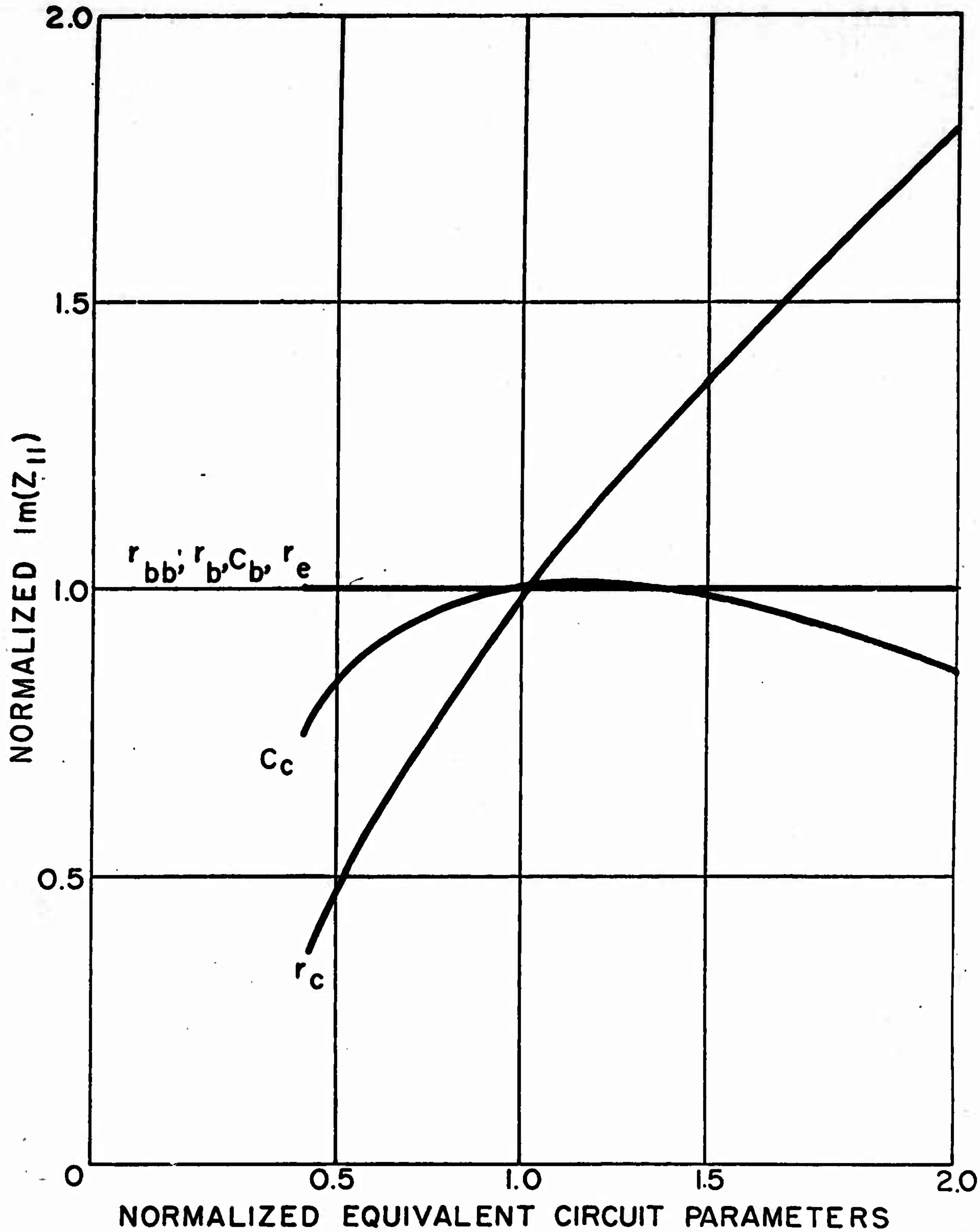


Figure 4. Imaginary part of Z_{11} versus equivalent circuit parameters. Common collector configuration.

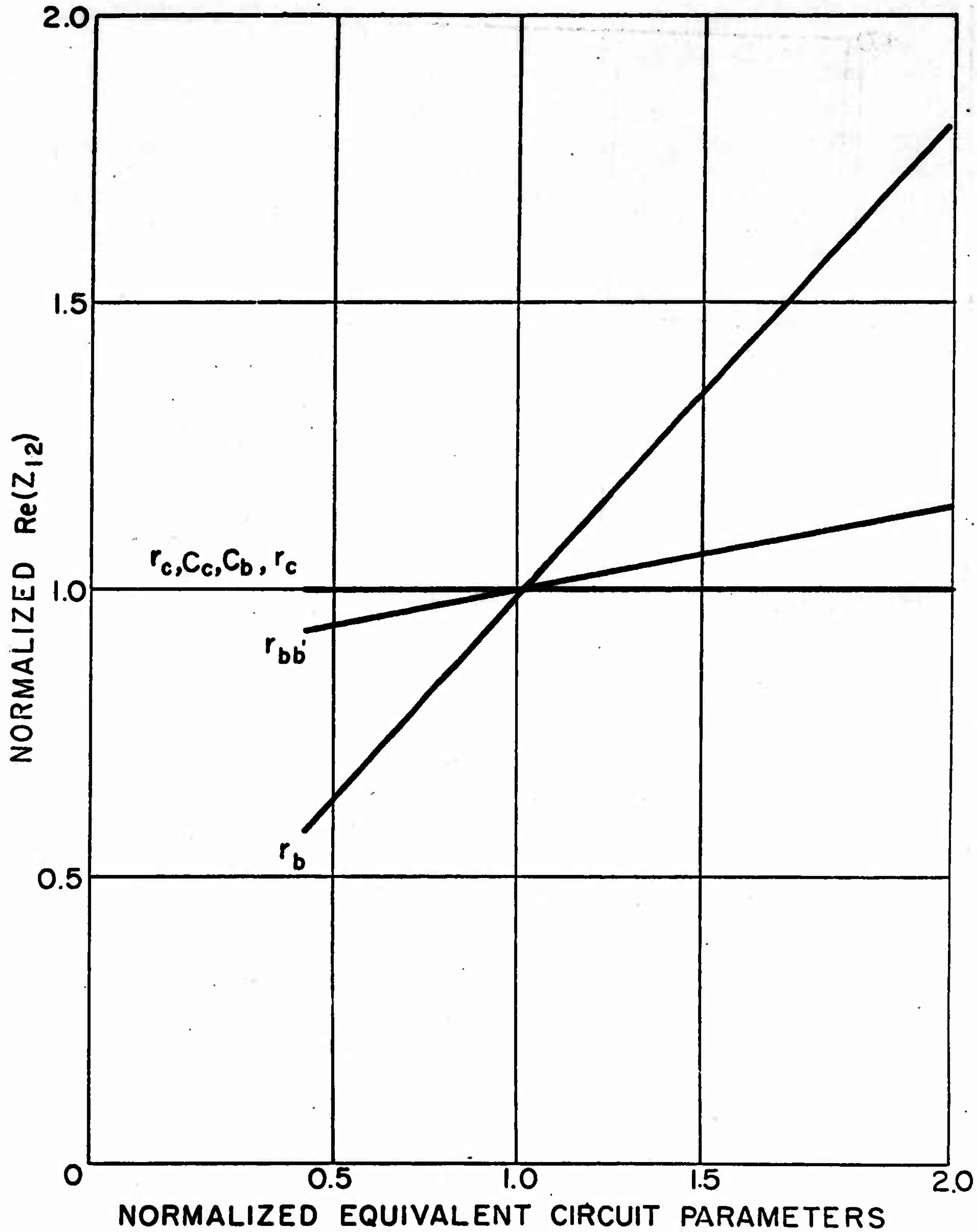


Figure 5. Real part of Z_{12} versus equivalent circuit parameters. Common base configuration.

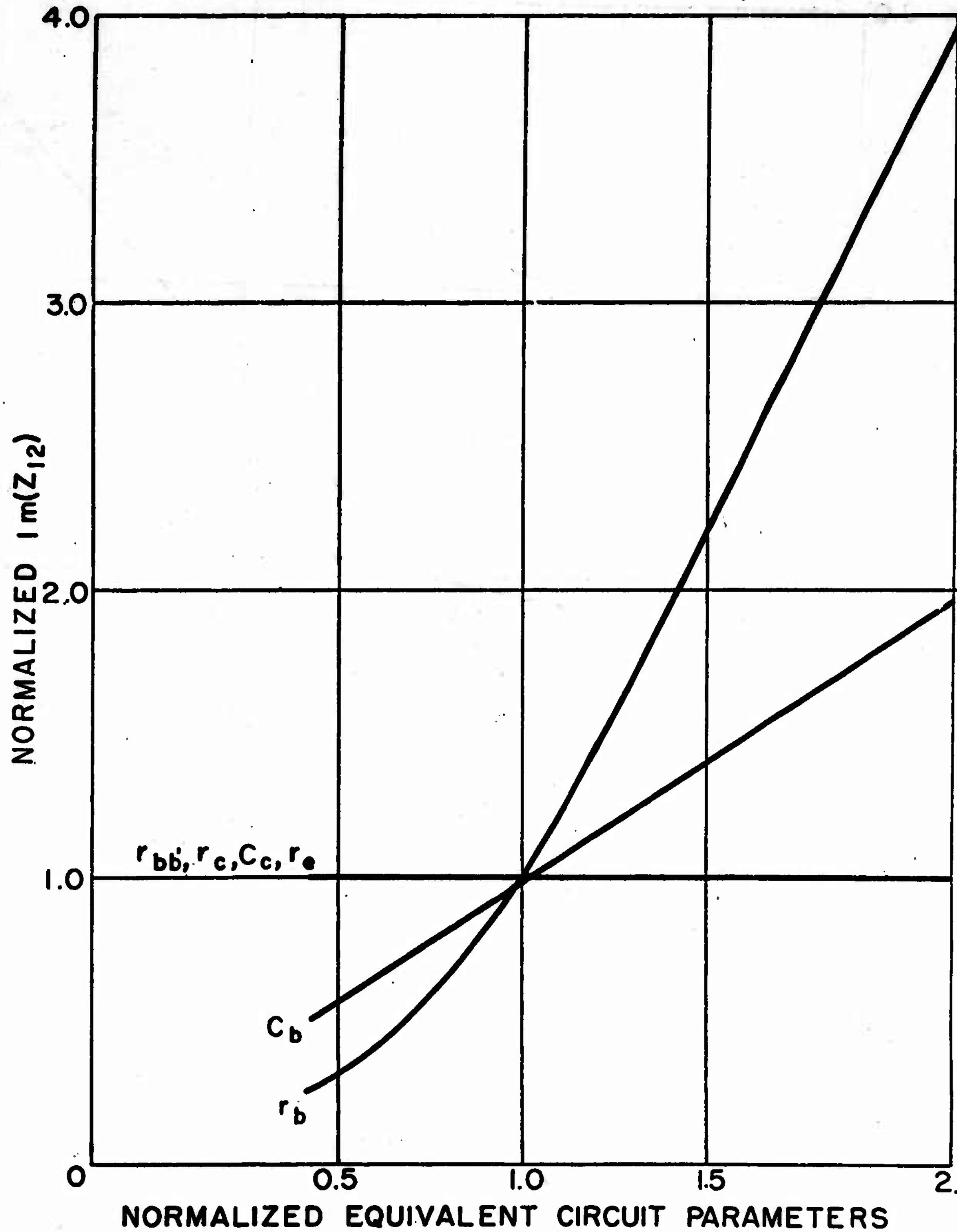


Figure 6. Imaginary part of Z_{12} versus equivalent circuit parameters. Common base configuration.

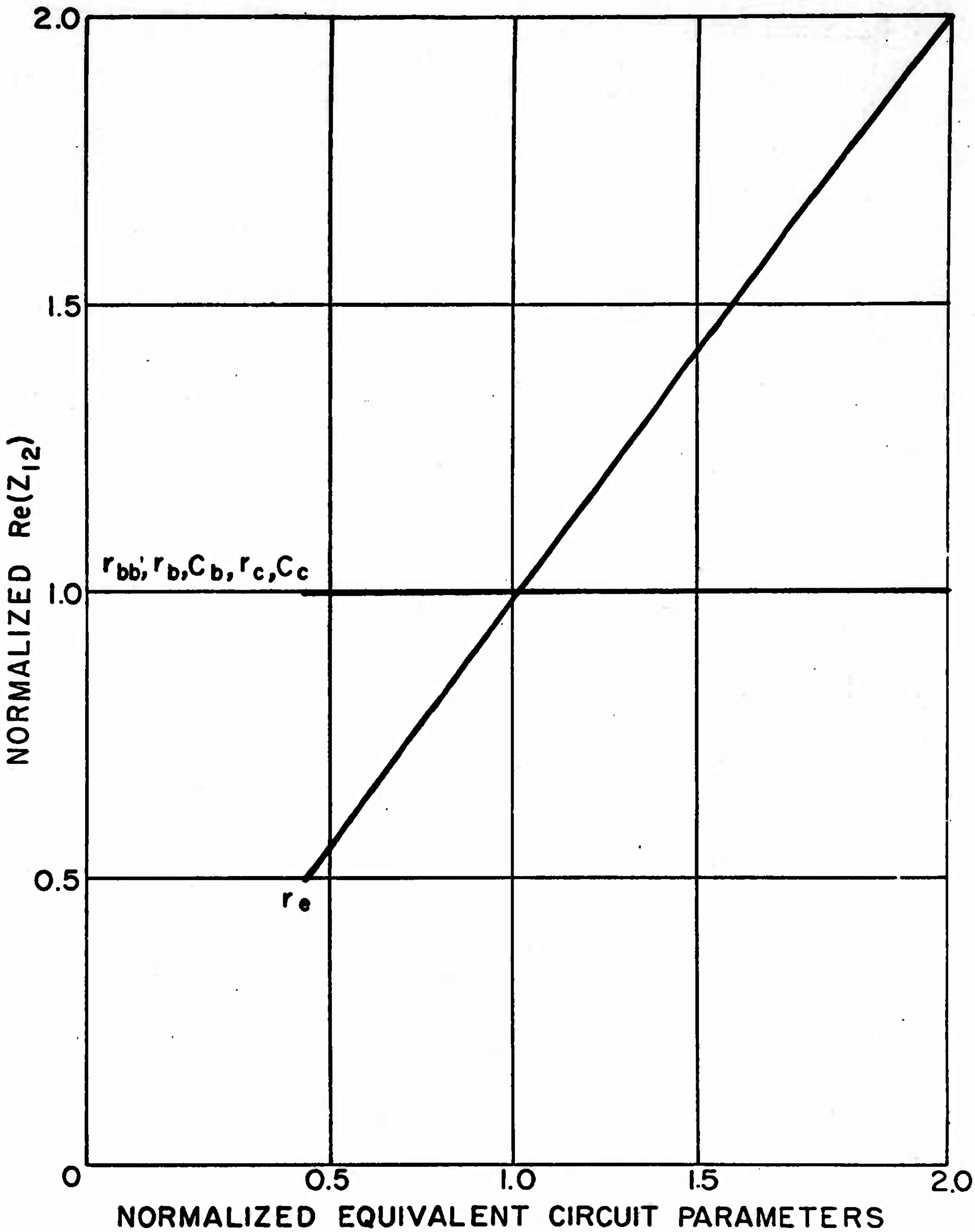


Figure 7. Real part of Z_{12} versus equivalent circuit parameters. Common emitter configuration.

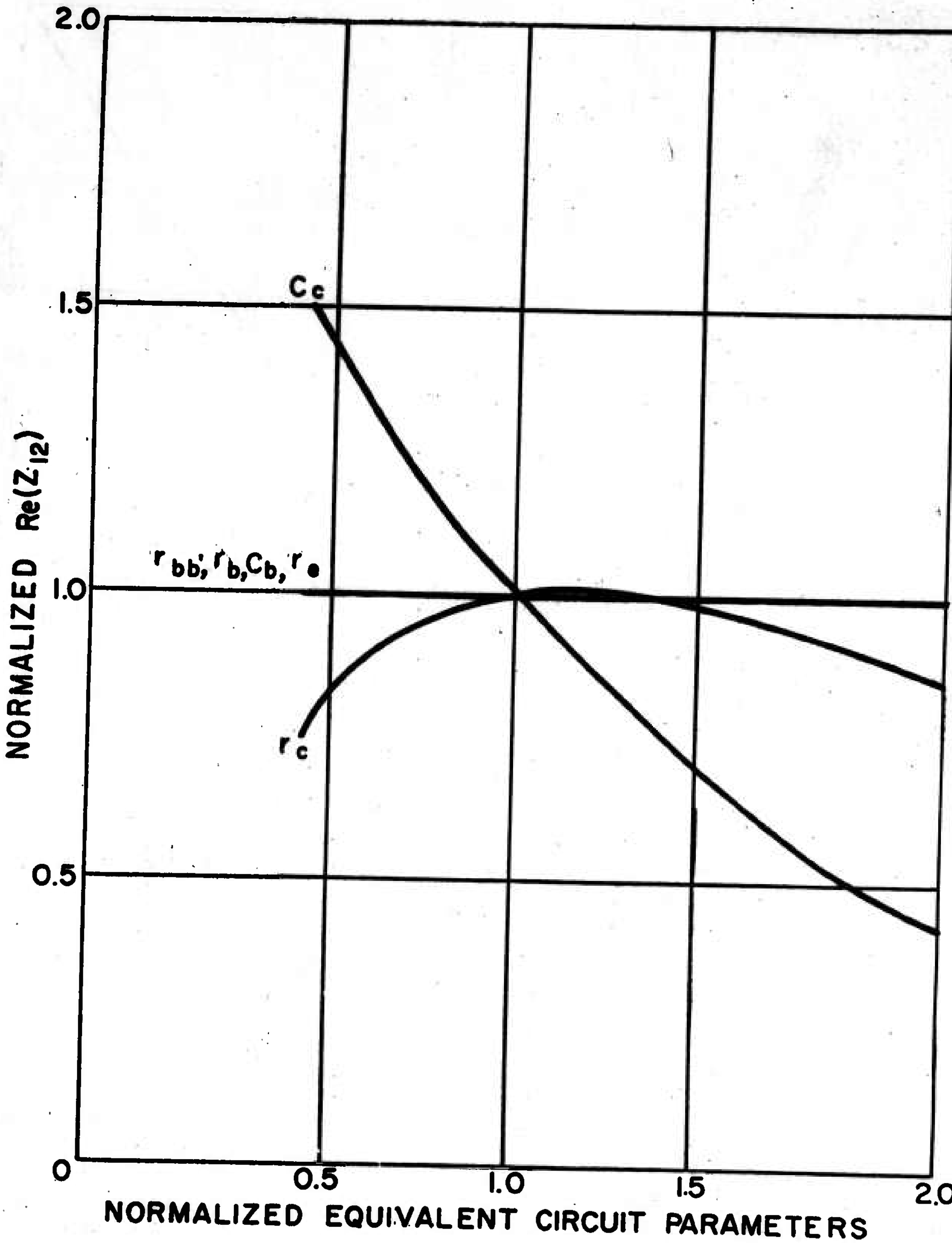


Figure 8. Real part of Z_{12} versus equivalent circuit parameters. Common collector configuration.

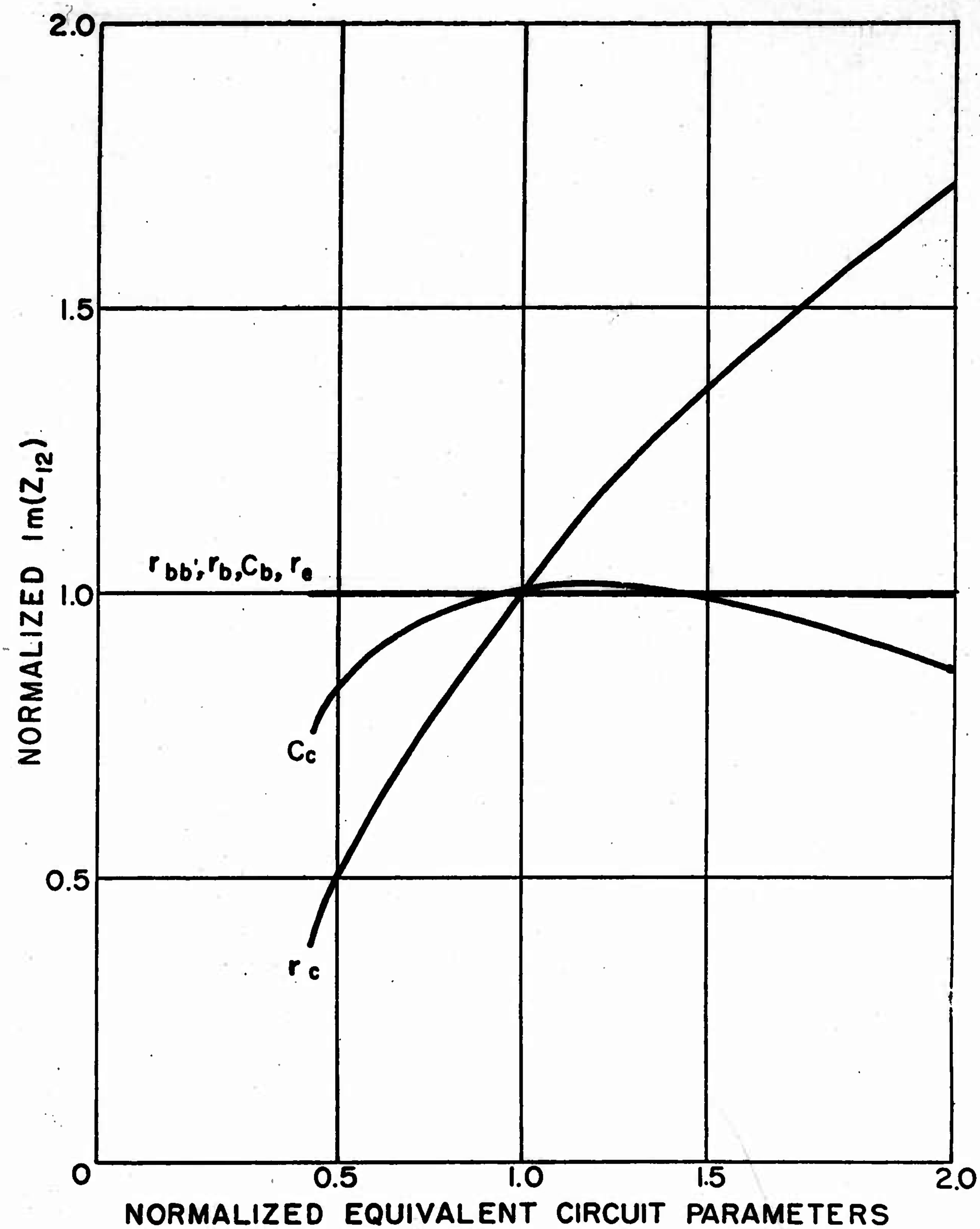


Figure 9. Imaginary part of Z_{12} versus equivalent circuit parameters. Common collector configuration.

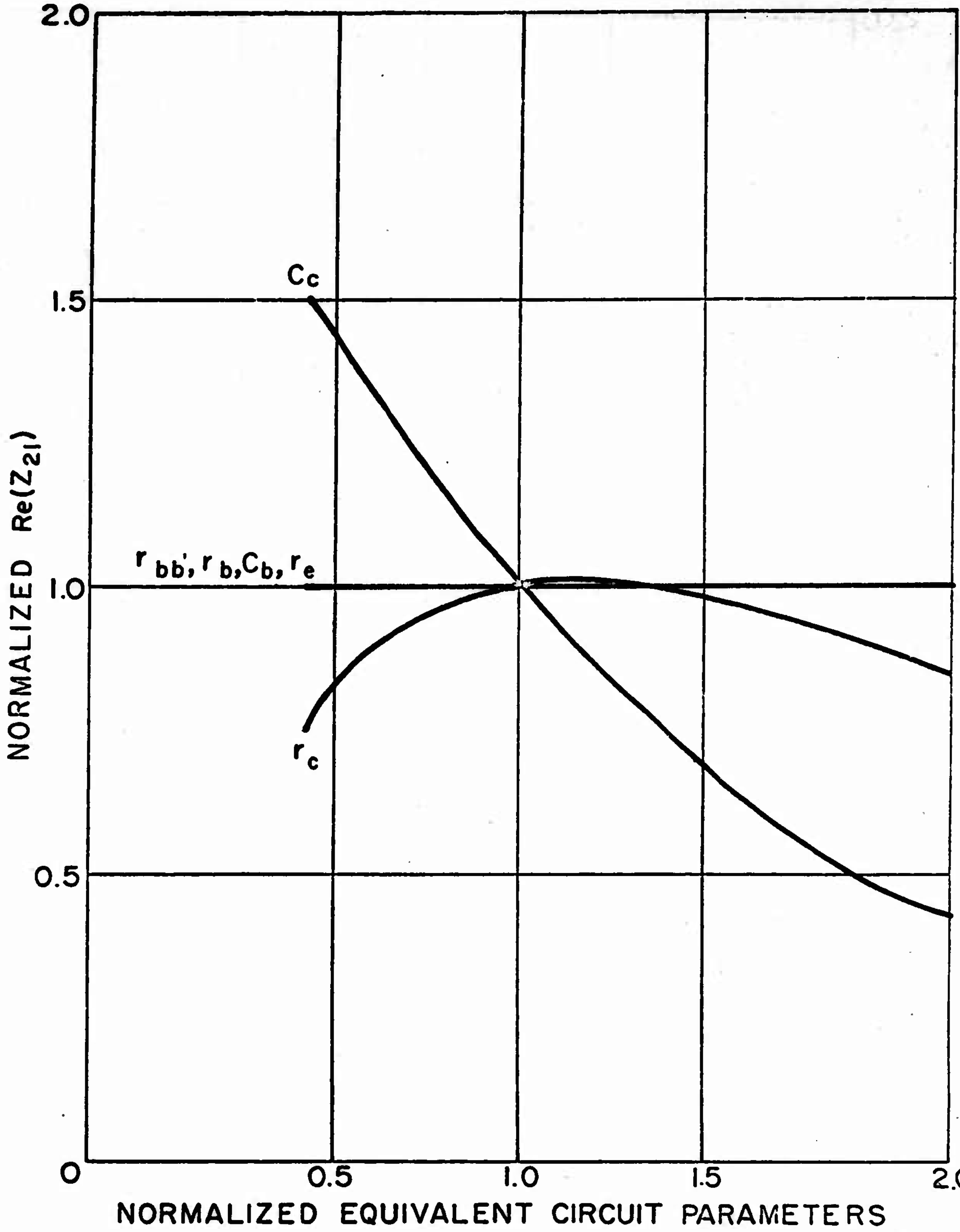


Figure 10. Real part of Z_{21} versus equivalent circuit parameters. Common base, collector, emitter configuration.

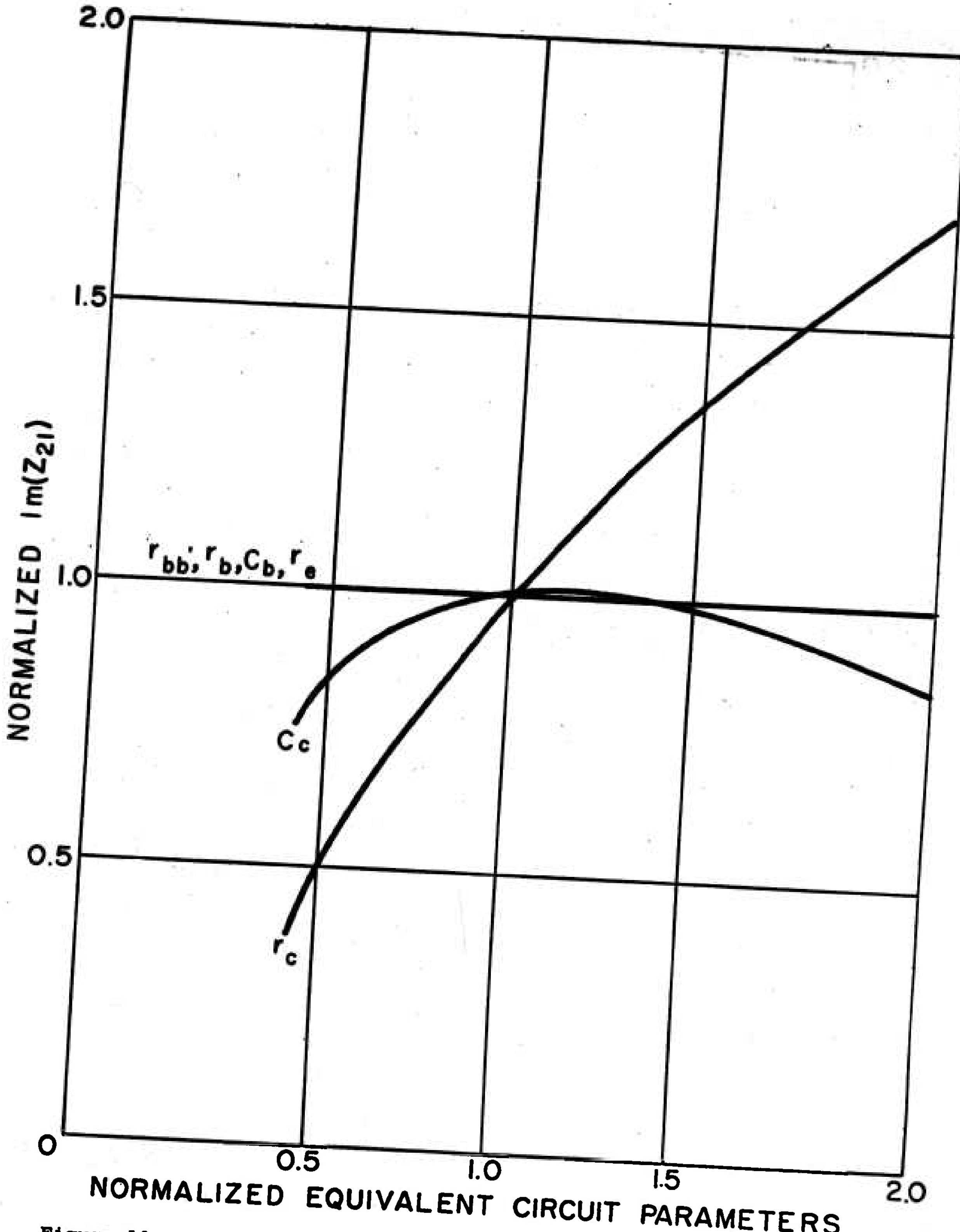


Figure 11. Imaginary part of Z_{21} versus equivalent circuit parameters. Common emitter, collector base configuration.

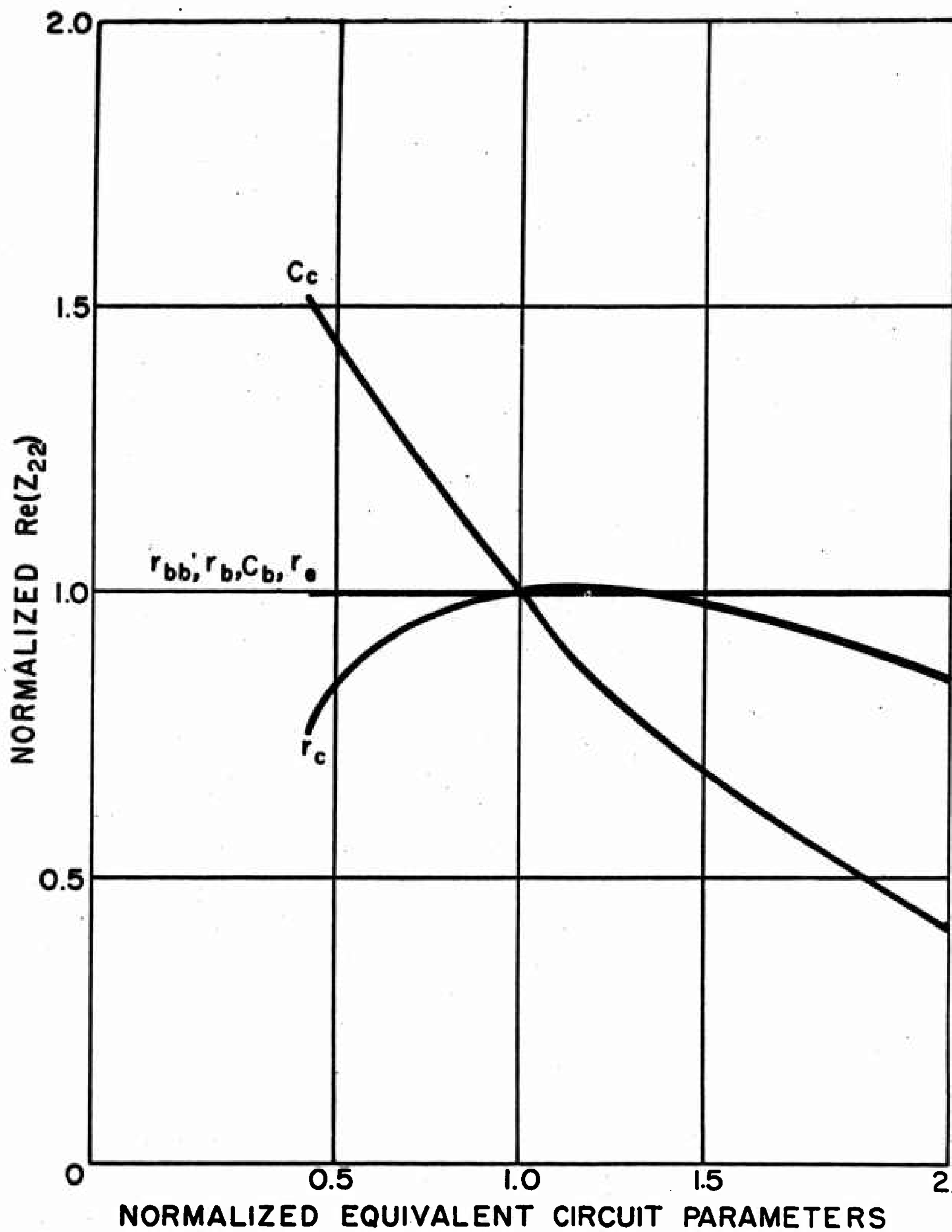


Figure 12. Real part of Z_{22} versus equivalent circuit parameters. Common emitter, collector, base configuration.

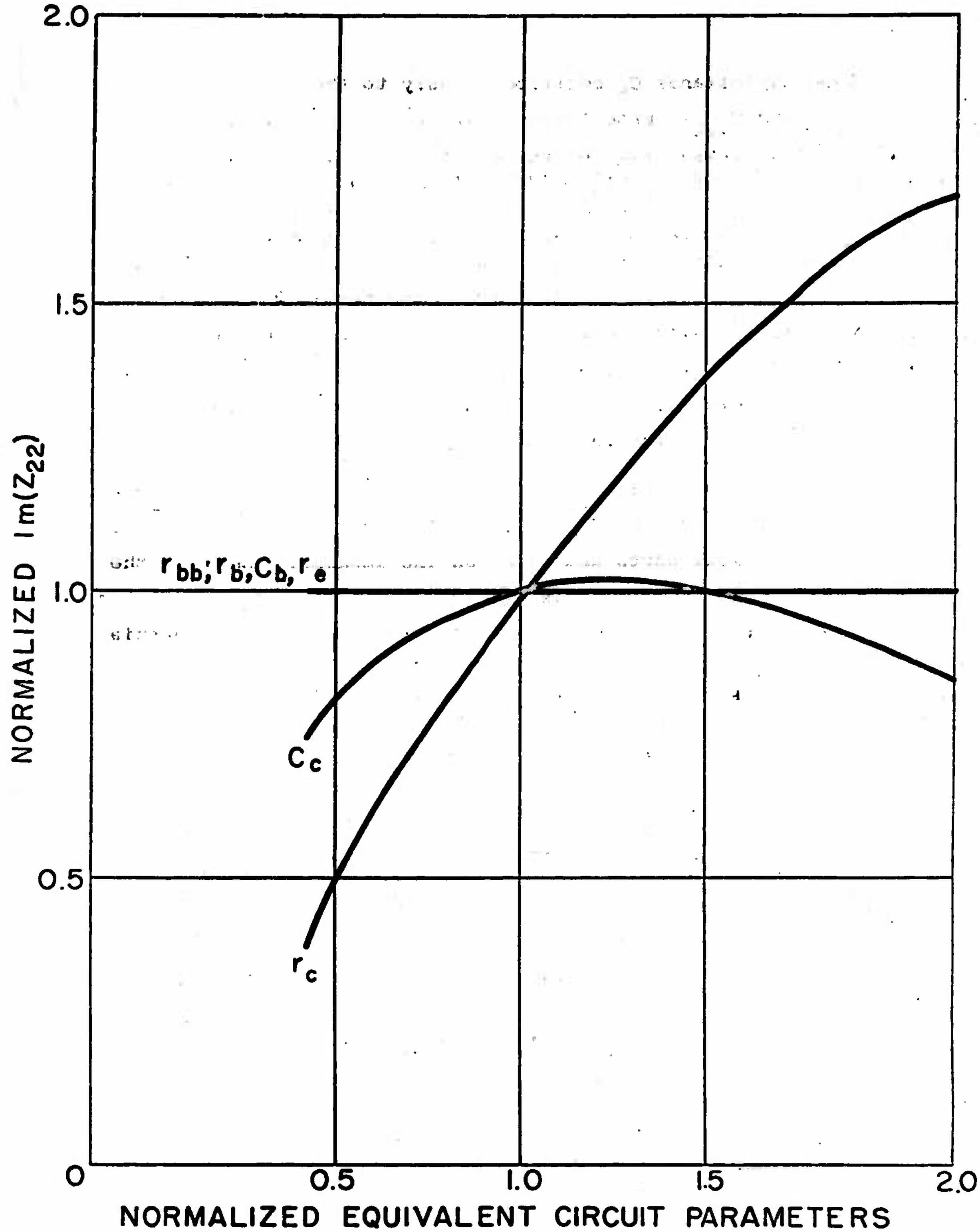


Figure 13. Imaginary part of Z_{22} versus equivalent circuit parameters Common collector, base, emitter configuration.

The base capacitance C_b contributes only to the imaginary components of Z_{11e}, Z_{11b} and Z_{12b}. From table II it is seen that these components are negligible in comparison to the real parts. Of course, as the frequency is increased, C_b will play a significant role on the terminal characteristics. If only the magnitude of the impedance is required, the base capacitance can be neglected. However, if phase angle information is needed C_b must be included. In the common emitter and common base input impedance the phase angle is approximately 2^o. By varying C_b from one-half to twice its nominal value, the phase angle could be changed from 1^o to 6^o respectively. Thus, the phase angle of the input impedance can be controlled to a certain extent by adjusting C_b, while its absolute value will remain constant.

In the common collection configuration, all four elements of the Z-matrix depend only on r_c and C_c. Of particular interest is the effect of r_c on the real parts and of C_c on the imaginary parts of the elements of the matrix. For values of r_c less than its nominal value, the real part of the impedance increase according to some exponential functional of r_c. As r_c reaches a value slightly higher than its nominal, the real part of Z, Re[Z], begins to decrease and it will continue to decrease as r_c increases. This rather unusual behavior can be traced to the expression

$$\frac{r_c}{1+(\omega C_c r_c)^2}$$

where for large values of r_c the term containing r_c^2 begins to influence the fraction more than the numerator r_c. The maximum value of the real part is given for a value of r_c slightly higher than its nominal value. Similarly, the imaginary components behave in almost identical manner for the variation in C_c. The term responsible for this effect of C_c is

$$\frac{\omega C_c r_c^2}{1+(\omega C_c r_c)^2}$$

Since r_c behaves in similar fashion in both the numerator and denominator, the imaginary terms of the Z-parameters, Im[Z], are affected by r_c according to some proportionality function. Thus Im[Z] increases with increasing r_c while C_c causes it to increase initially and then reverse slope. The points of inflection were calculated after the data indicated that either halving or doubling r_c and C_c the impedance was less than that of the nominal values.

A few remarks can also be made as to the dependence of the absolute value of the Z-parameters on r_c and C_c. It is obvious that any increase in the collector capacitance will cause the impedances which depend on C_c to decrease, perhaps slowly in the beginning but rather rapidly as C_c attains values higher than its nominal value. The effect of r_c, however, is not as easily predictable. For values of r_c less than nominal, the absolute value of the impedance will increase since both the real and the imaginary parts increased with r_c. As the value of r_c becomes much larger than nominal, the real part begins to decrease while the imaginary part continues increasing. To the extent for which information is available, $|Z|$, which depends almost equally on the real and imaginary parts (table II), will continue to increase with increasing r_c but at an ever-decreasing rate. The phase angle will experience a rapid increase since the imaginary part increases and the real part decreases with r_c. It should be kept in mind that all the parameter variations are for a frequency of 1.0 kc. For any other frequency the curves might assume a completely different form.

4. ACKNOWLEDGEMENT

The author wishes to thank Mr. George Kambouris for many helpful suggestions regarding this investigation.

5. REFERENCES

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2. DOFL TR-502, "Transistor Parameter Variations in the VLF Range", G. Kambouris and H. Morris, 1 September 1957.

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